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**Review of Risk and
Reliability Methods for
Aircraft Gas Turbine Engines**

Joanna Kappas

DSTO-TR-1306

20021025 374



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Airframes and Engines Division
Aeronautical and Maritime Research Laboratory

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ABSTRACT

Risk and reliability assessment of aircraft gas turbine engines for the evaluation of component failure has received increasing interest in the last few years, fuelled by the greater appreciation of stochastic models and the concern for airworthiness issues. This report reviews the current status of probabilistic methods available for the risk and reliability assessment of gas turbine engines and the potential benefits of their implementation in the military environment. The definition of acceptable risk of failure in the military standards and the current relevant activities in the U.S which are of particular interest to the RAAF, are also discussed.

RELEASE LIMITATION

Approved for public release

AQ FOI-01-0111

Published by

*DSTO Aeronautical and Maritime Research Laboratory
506 Lorimer St
Fishermans Bend, Victoria 3207 Australia*

Telephone: (03) 9626 7000

Fax: (03) 9626 7999

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AR-012-320

May 2002

APPROVED FOR PUBLIC RELEASE

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Executive Summary

The risk and reliability assessment of aircraft gas turbine engines using probabilistic methods for the analysis of component failure is a recent development for military aircraft. The increased interest has resulted from greater appreciation of stochastic models and the current concern for airworthiness issues of aging aircraft.

There are several current activities in the aerospace industry that aim to reduce the current gap between the risk assumptions of the safe life methodology, the military standards and the operational requirements for aircraft gas turbine components. These include the FAA Titanium Rotating Components Review program, the U.S. Air Force Blade Optimisation program and the annual FAA/Air Force/NASA/Navy workshops on the application of probabilistic methods to gas turbine engines. The aim of these collective activities is to increase the capability of the aerospace industry in probabilistic methods for the risk and reliability assessment of critical engine components.

This report has reviewed the issues that drive the change away from a deterministic towards a stochastic approach in the design and analysis of engine component lives. The methods and programs available have been addressed. Of particular interest are the modular programs resulting from collaboration between research organisations and the OEMs. Some of these programs incorporate an inspection maintenance approach and thus have potential use when a Retirement for Cause or Damage Tolerance approach is adopted for engine components.

The dependence of risk and reliability assessment on the availability of engine data has been discussed. This is a fundamental issue for gas turbine engines where thermal, manufacturing and machining effects on material properties need to be taken into consideration in understanding the failure modes. For gas turbine engines with military applications, databases describing material properties of alloys generally belong to OEMs and are not widely available, adding further difficulties in determining the probability density functions necessary for accurate risk and reliability assessments.

The review has referred to the military standards in determining what is the acceptable risk of failure of gas turbine engine components. Although estimates of the acceptable risk are given in UK DEF STAN 00-971, other standards lack quantified risk limits.

It is evident that risk assessment of gas turbine engine components using probabilistic methods is attracting considerable interest and effort in the military aviation community and a move away from the use of deterministic methods towards the implementation of stochastic approaches in engine lifing is in progress.

The report has highlighted areas in need of further research, including the effects of multiple failure modes incorporating creep and oxidation effects on risk of failure for engine components, and the development of engine system risk models.

Authors

Joanna Kappas

Airframes and Engines Division

Joanna graduated from Monash University in 1995 with a B.Science /B.Mech.E (Hon). Following her work at Wind Tunnel Consultants on structural wind loading, she obtained a M.Eng.Sc.(Research in Foundry Industry with Holden Engine Company) from Monash University. In 1999 she commenced work with AED and is currently involved in mission analysis and usage monitoring for engine components and risk assessment of propulsion dynamic components.

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NOTATION

a_i	Rate of Defect Occurrence in Zone I
ADF	Australian Defence Force
β	Weibull shape parameter (slope)
CARES/Life	Ceramics Analysis and Reliability Evaluation of Structures
CMC	Ceramic Matrix Composites
D	Region of failure
EFA	Eurofighter Aircraft
ENSIP	ENgine Structural Integrity Program
FEA	Finite Element Analysis
FAA	Federal Aviation Administration
FMECA	Failure Modes, Effects and Criticality Analysis
FORM	First Order Reliability Method
$f(x_1, \dots, x_n)$	Joint Probability Density Function
g	Limit State Function
L	Life
L_{ref}	Reference Life
LCF	Low Cycle Fatigue
M_p	Material Property
MCS	Metal-Composite Structures
MMC	Metal-Matrix Composites
MVFO	Mean Value First Order
NDE	Non Destructive Evaluation
NERF	Numerical Evaluation of Reliability Functions
NESSUS	Numerical Evaluation of Stochastic Structures Under Stress
OEM	Original Engine Manufacturer
P_f	Probability of Fracture
PFM	Probabilistic Fracture Mechanics
PMC2001	First Annual Probabilistic Methods Conference
POD	Probability Of Detection
PRA	Probabilistic Risk Assessment
PROF	PRObability of Fracture
PSAM	Probabilistic Structural Analysis Methods
R&R	Risk and Reliability
RSM	Response Surface Method
S	Probability of Survival

S_c	Probability of Survival of Engine Component
S_{ref}	Reference Stress
SORM	Second Order Reliability Method
SwRI	Southwest Research Institute
τ	Shear Stress
τ_{ref}	Reference Shear Stress
V	Volume
V_{ref}	Reference Volume
X_1	Stress Multiplying Factor
X_2	Life Scatter Factor
$x_1 \dots x_n$	Failure drivers
Z	Random Variable

1. Introduction

An important issue of concern to the ADF, like other military aircraft operators, is the element of risk in the operating environment due to engine component failure. Specific concerns are:

- What is the risk of flying an aircraft,
- What is the acceptable flight failure limit, and therefore
- How can the risk of flying an aircraft be managed by the ADF?

Probabilistic methods can be used to address the first issue, an important step towards risk management.

Military aircraft engine components have traditionally been lifed by the Original Equipment Manufacturer (OEM) using the safe life method. The risk of continuing operation of an engine component beyond its safe life has been questioned in circumstances of high cost of component replacement or of component scarcity. The quantification of risk is desirable for decision making at operational level. It would also allow a means of comparison with the risk of failure requirements described in the military standards for engines. Risk assessment can contribute to the implementation of Retirement for Cause for engine components by defining the inspection interval for an acceptable risk level. There is therefore great potential in the implementation of risk and reliability methods for the management of military engine components.

Risk is, in this context, the probability of failure within the service life of the engine/component while reliability is the probability of the engine/component not failing during service. The term Risk and Reliability (R&R) assessment will be used in this report to describe the determination of the probability of failure /survival of an engine component. The methods examined in this review are mostly probabilistic and the term probabilistic assessment will be used interchangeably with R&R.

This report reviews important issues arising from the application of R&R methods to gas turbine engine components. The contemporary view on the safe life method will be addressed and the move from deterministic to probabilistic approaches in the design/lifing of engine components will be discussed. Important activities initiated by the FAA and the U.S Air Force will be presented.

Computational methods are briefly described. The report places emphasis on the data input requirements for R&R programs and discusses the problems due to scarcity of data and means of overcoming them. Programs developed specifically for gas turbine engine components and generic programs of potential benefit are listed and examples of their application are given. Areas where limited information is available in the literature are listed.

This report aims to describe not only the status of risk and reliability assessment of gas turbine engines but also the trends in the aircraft industry and the U.S. military that may affect ADF management of engine components in the future.

2. Need for Risk and Reliability Analysis

As Cruse et al (1997) state, the safe life design philosophy, led by companies such as Pratt & Whitney and Rolls-Royce, has been the basis of aircraft engine design today. A brief history of the development of the safety factor concept, where a safety factor is applied to the design loads to provide a safety margin in safe life, is given by Goldberg and Verderaine (1999). The safe life philosophy has been used by OEMs to determine the number of flight cycles before cracking can be expected at a low probability of occurrence, and to use that minimum life to certify the aircraft engine components. Engine components are retired at a life that has a one in a thousand chance of a 1/32" crack developing in that time.

A number of concerns have led to the investigation of alternative approaches to determining engine disk life. In one instance it was found that stresses in engine disks had not been adequately calculated in the early design periods, before finite element methods were available, and several disks failed short of their initial design life [Cruse et al 1997]. In addition, the inherent cost associated with the safe life approach due to the wasting of expensive uncracked components became an issue. The Damage Tolerance and Retirement for Cause methods were examined by the U.S. Air Force. The first was initially found inappropriate for engine material alloys, setting an excessively short inspection interval. The latter exposed issues associated with probabilistic fracture mechanics and the liability of missing a crack at inspection [Cruse et al 1997]. Since 1984, Damage Tolerance forms an integral part of the ENgine Structural Integrity Program (ENSIP), contained in MIL-STD-1783. There is now a program in the U.S. to incorporate guidelines for application of probabilistic methods for gas turbine engines in the ENSIP by 2006 [Brown 2001].

The philosophy and implications of introducing Damage Tolerance concepts into the design and use of critical engine components were discussed by AGARD in 1985 [N.A.T.O 1985]. The Damage Tolerance philosophy was stated to offer potential cost savings of considerable magnitude when compared with a 'safe-life' approach, if only it can be implemented with an assurance that current safety standards will not be prejudiced. It was estimated *that over 80% of engine disks have 10 or more low cycle fatigue lives remaining when discarded under 'safe-life' rules.*

Development of Retirement For Cause or "fly to a safe crack" approach allows periodic inspection of each disk so that the only disks that are retired are ones with detected cracks [Bartsch 1985, Ohnabe et al 1985]. This method requires that an optimal inspection interval should be determined. Probabilistic methods can be used to determine the fatigue reliability of engine components under scheduled inspection maintenance and also the optimum inspection interval for a certain risk level.

For commercial gas turbine engines both Damage Tolerance and Retirement For Cause methods have been adopted. During early 1980's, the FAA accepted a Probabilistic Risk Assessment (PRA) using an in-service Retirement For Cause program, the success of which lead to the adoption of PRA by the FAA for commercial engine disks [Cruse

et al 1997]. When an in-service cracking problem occurs, FAA requires probabilistic fracture mechanics and risk assessment so that no commercial engine is flying with a significant crack size, rather than grounding all engines until the components are replaced.

Military authorities, including the ADF, have generally been slow in adopting a similar approach and prefer to ground the fleet and replace components following in-flight cracking. As Koul et al (1985) describe, lack of replacement components can initiate a damage tolerance approach. Such an example is the J85-CAN40 engine where the reduction of the safe life limits by the OEM resulted in insufficient numbers of replacement components being available, and a damage tolerance approach had to be implemented.

The question of validity of the manufacturer's analytical life predictions for the safe life of engine components has been raised by many authors but has not been answered adequately [Mahorter et al 1985b, Singpurwalla 1988]. Researchers that utilised spin pit testing to experimentally obtain the life of components compared their findings with the safe life predictions which gave antithetic and inconclusive results [Mahorter et al 1985b, Melis and Zaretsky 1999].

The conservative lifing of OEM analytical models was investigated in a study by Mahorter, London, Fowler and Salvino (1985) that produced spin-pit results for time to crack initiation for compressor disks. The accuracy of the safe life prediction for military applications was questioned when only a very small proportion of engine components 'fail' at the predicted life.

Singpurwalla (1988) states that aircraft manufacturers' analytical life models for engine components have been quoted to produce conservative predictions of up to 50% for two main reasons, the desire for increased safety and the drive for increased number of components sold. He states that OEM analytical models are company confidential and based on a large number of specimen tests of material properties; one example quoted 300 specimens. It is certain that OEM material databases would be of great benefit to research in component failures and risk assessment of engine components, as also concluded in the DSTO study of the TF30 engine [Tong and Kappas 2002].

An additional element that affects the turbine rotors' life is the material and manufacturing anomalies, which occur infrequently and most likely are not accounted for in laboratory specimens and full-scale component testing that is used to determine the safe life. These undetected anomalies have lead to several incidents including the loss of the DC-10 at Sioux City in 1989 [AIARI Sub-Committee 1997].

According to Berens (1996), today's aging military aircraft are exceeding their designed life and are being used in their wear out phase. He promotes R&R methodology as one possible tool for making cost effective decisions about inspections, repair and life extension of military aircraft.

Cole (1998), among others, challenges the validity of the "constant failure rate" theory, and the expectation that failures on a fully developed engine come only from the 'random' region of the bathtub curve. He therefore questions the reliability targets

defined in the Failure Modes, Effects and Criticality Analysis (FMECA), where the system failure rate is determined as a simple sum of component failure rates. He refers to the example of LCF failures where there is an observed increasing failure rate with time.

As Melis and Zaretsky (1999) state, deterministic methods assume that full and certain knowledge exists for service conditions and material properties. Deterministic failure mode analysis has been described by Moore, Ebbeler and Creager (1990) as either non-representative of the conditions during the flight, too conservative by the application of safety factors, or inaccurate due to limited information and uncertain knowledge. They further state that the inaccuracy of engineering models used in deterministic analyses to describe sometimes-complex failure phenomena may produce uncertain results [Moore et al 1990].

As explained in [Lust and Wu 1998, Xiaoming et al 1998], the non-deterministic nature of fatigue failure stems from material and geometric tolerance uncertainties, environmental conditions, uncertainty in service loading and unintended use, as well as variations in manufacturing and assembly processes. Probabilistic assessment of failure risk allows the quantification of the uncertainties or variability affecting failure of a structure or a component.

A number of researchers discuss probabilistic methods that are applicable to situations when information describing failure is difficult or expensive to acquire. The aerospace industry has acknowledged the significant influence of uncertainties present in loads, material behaviour and geometric configuration, and has developed probabilistic structural analysis methods, some of which are generic in nature and can be applied to aircraft and engine reliability [Moore et al 1990, Shah et al 1992].

As described by Moore, Ebbeler and Creager (1990), the urgent need for improved approaches to managing failure in launch vehicle propulsion systems has been addressed by the introduction of probabilistic methods for quantitatively assessing failure risk [Moore et al 1990]. The authors mention the limitations of past practices in engineering areas that used judgemental evaluation based on limited test experience and deterministic engineering analysis. The aircraft industry today is in a similar position and is slowly moving into probabilistic assessment of failure.

One area where probabilistic methods have been adopted early in the design phase is in the use of ceramic materials. The recent development of ceramic components for gas turbine engines required a departure from the usual deterministic design philosophy due to the large variability in ceramics' material properties. Consequently, probabilistic methods have been readily adopted for ceramic component reliability [Powers et al 1992]. The introduction of R&R methods at the design and development phase is being increasingly recommended in the aircraft and aerospace industry because of the inherent uncertainties and the cost advantages of reduced conservatism [Moore et al 1992b, AIARI Sub-Committee 1997].

Moore, Ebbeler and Creager (1990) mention that parallel to the increasing need to implement probabilistic methods in R&R analysis, there is still the need to determine the origins, mechanisms and consequences of known failure modes. Understanding

the nature of failure modes provides a foundation for failure models and a small number of studies have been conducted with the aim of achieving that for gas turbine engines [Cruse et al 1997, Leverant et al 1997].

Therefore the need for R&R assessment of gas turbine engines is not only driven by alternative lifing methods, Damage Tolerance and Retirement For Cause, but follows the developments in commercial engines and aerospace industries which are replacing deterministic methods with stochastic/probabilistic approaches to engine lifing and engine failure assessment. This change in attitude became obvious following the 1989 Sioux City accident, when the Federal Aviation Administration (FAA) Titanium Review Team recommended the introduction by industry of additional damage tolerance concepts to reduce the rate of uncontained rotor events related to material anomalies [AIARI Sub-Committee 1997]. This recommendation has spurred a great effort towards a fracture mechanics-based probabilistic analysis of all new critical gas turbine engine components. It has added Damage Tolerance to the OEM existing design and life management process acknowledging the effect of inherent, manufacturing and maintenance induced material anomalies. Although it does not replace the Safe Life method, it introduces probabilistic methods to complement it. Similarly the Blade Optimisation Program running in parallel to the FAA program, was initiated in 1990 by the U.S. Air Force Research Laboratory with engine OEMs to implement probabilistic methods in the design of gas turbine components in order to reduce weight and increase durability. The results of the program will be incorporated as guidelines in the ENSIP. Such programs show the increased interest and use of R&R in the last decade, supported by the military and aviation authorities.

2.1 Statistical Failure Assessment and Certification

Certification of flight readiness has, in the past, used certification rules with no formal rationale based on statistics or engineering analysis, for both commercial and military sectors [Moore et al 1990]. Such an example in aircraft engine components is the LCF life prediction based on the initiation life [Mahorter et al 1985b, Cruse et al 1997]. The engine part life, also called B.1 life, is the LCF life defined by a crack growing to 1/32 inch in any of the critical areas with a one in a thousand chance. The accuracy of the LCF life times and the appropriateness of the B.1 level have been questioned. The wastage of non-cracked parts at the end of their LCF life also raises the question of expenses [Mahorter et al 1985b].

Cole (1998) describes the developments in statistical failure analysis of aero gas turbines. The initial requirements for certification of aircraft originated for the Concorde and the Tornado in the 1960s. These have expanded to the safety and reliability levels that have become contractually binding for the EJ200 in the Eurofighter in the 1990s. The use of the 'hazard matrix' for safety, according to the author, is increasingly becoming mandatory in American and UK military contracts. Cole (1998) lists as examples of its use, the Pegasus in the Harrier and Tornado military aircraft. An example of the hazard matrix for safety can be seen in Figure 1, showing

the bands for quantitative probability and qualitative severity levels as set by DEF STAN 00-971 (1987). Clearly, the implementation of such a matrix requires means of quantifying the probability of failure.

Probabilistic failure risk assessment necessitates the use of acceptable risk criteria for the evaluation of numerical results. Most reports in this review describe results for assessment of probability of component failure but the underlying question of 'what is acceptable' is not always answered. Often in the literature, the acceptable risk of 10^{-6} is mentioned, without providing the source or reason for the selection of this criterion [Berens 1996, Szekely et al 1998]. During the First Annual Probabilistic Methods Conference it was strongly stated that probabilities acceptable for failure limits need to be defined [Brown 2001, Rogers 2001].

A unique example of setting risk targets for titanium rotating components is the recent study generated by the FAA Titanium Rotating Components Review following the 1989 Sioux City accident. Design Target Risk (DTR) values were set by an industry committee to determine the acceptability of new titanium rotating design components. The DTR values were determined at component and engine levels, in terms of events/part cycle, so that component re-design would be implemented when these values are exceeded [AIARI Sub-Committee 1997].

2.1.1 Engine certification

UK and US Military standards provide guidelines for engine certification. USAF MIL-STD-1783 (1984) describes requirements for ENSIP, safety and durability. It includes safety factors as part of design development and verification and Damage Tolerance requirements (USAF). It does not specify acceptable levels of probability of failure other than those implied through the safety factor design method.

US MIL-STD-882C (1993) describes the process by which a *hazard risk assessment matrix* is produced for military systems. The standard states that decisions regarding hazard severity, hazard prioritisation and acceptable levels of probability of failure must be agreed between the military party and the contractor assessing the failure of the military system. This reference only provides general guidelines in producing the risk assessment matrix and an engineering assessment is then necessary to quantify the acceptable risk criterion.

The UK DEF STAN 00-971 (1987) for aircraft gas turbine engines provides guidelines for engine certification in terms of reliability. It states that a Failure Mode and Effect Analysis (FMEA) for the complete engine should "assess the likely consequences of all failure modes that can be reasonably expected to occur". The failures that could result in a severity of effect greater than minor should contain an estimate of the probabilities of occurrence of those failures, in terms of 'reasonably probable', 'remote' or 'extremely remote'.

The classification of the failure modes, effects and acceptable risk levels described in ANNEX B of DEF STAN 00-971 (1987), is summarised in Figure 1, for military non-passenger aircraft. For passenger carrying transport aircraft the probability of failure limits are tighter, as greater loss of life is at risk.

The effect of failure is classified as minor, major and hazardous and the last of these is allowed an extremely remote chance of occurring. For engine certification of non-passenger aircraft, it is acceptable that for the likelihood of a hazardous event, such as non-containment of energy debris, *the probability of occurrence should not be greater than 10^{-6} per hour of flight*. It is the effect (hazardous) therefore and not the cause (eg disk fracture) that is used for assessing certification. For an engine system, a number of components, and therefore critical locations, can contribute to an individual failure of hazardous effect; hence the calculation of the system risk limit of 10^{-6} per hour of flight for a particular engine can be a difficult exercise. The issues involved in determining engine system risk and applying the DEFSTAN 00-971 criteria are further discussed in a report by Kappas and Antoniou (2002).

DEFSTAN 00-971 Classification of effect of engine failure	Probability of failure per flight hour		
	Reasonably Probable (once or several times during life of aircraft) 10^{-3} to 10^{-4}	Remote (several times during total life of number of aircraft) 10^{-4} to 10^{-5}	Extremely Remote (unlikely to occur during total life of number of aircraft) 10^{-5} to 10^{-7}
Hazardous effect Death or serious injury of occupants Large reduction in safety margins Crew unable to perform tasks accurately or completely	UNACCEPTABLE		
Major effect Injury to occupants, Significant reduction in safety margins, reduction in ability of aircrew to perform under adverse operating conditions		ACCEPTABLE	
Minor effect Physical effects but no injury to occupants Slight reduction of safety margins Slight increase in workload			

Figure 1 Hazard matrix showing the classification of effect of engine failure [UK DEF STAN 00-971 1987]

3. Probabilistic Methods

According to Lepikhin, Moskvichev and Doronin (1998), there are three ways to assess risk:

1. Data-base approach, extrapolate past failure data,
2. Probabilistic engineering analysis with probabilistic fracture mechanics (PFM) as a subset,
3. Combined database and PFM.

Probabilistic engineering analysis may include the probabilistic life prediction of crack initiation, not covered by PFM. The programs considered in this review, implement PFM either by using stochastic models of crack growth or by using deterministic crack growth models with probability density functions describing the variability in component life and material properties.

3.1 Probabilistic Engineering Analysis

3.1.1 Probabilistic fracture mechanics

The probabilistic fracture mechanics approach involves probabilistic assessment of fatigue crack growth from initial to critical size, assuming an initial crack size and ignoring initiation life [Yang et al 1983, Graham and Mallinson 1984].

Various probabilistic fracture mechanics models described in the literature introduce stochastic parameters into empirical crack growth models [Yang et al 1983, Ohnabe et al 1985, Provan 1987, Min and Xiong 1992, Moore et al 1992b, Cruse et al 1997, Orisamolu and Luo 1997]. Cruse et al (1997) describe a stochastic interpretation of the Paris law where the Paris exponent n is constant and the Paris constant C is lognormally distributed. Other researchers use the Paris law with both n and C as jointly distributed random variables [Provan 1987, Min and Xiong 1992]. It is clear that material crack growth data has to be available to justify the choice of the type and value of the distribution function used for the stochastic parameters in the empirical models.

An example of how a deterministic model can become stochastic is described by Ohnabe et al (1985), where a random variable Z is introduced in the lognormal crack growth rate model (equation 1). Z is the product of random variables that contribute to the scatter in crack growth (equation 2). The random variables H_1, H_2, H_3, H_4 , describe variability in material, crack geometry, crack modelling and crack growth damage, and S describes the variability in stress due to service loads and the temperature profile and are each represented by lognormal distributions.

$$\frac{da}{dt} = ZQa^b \quad (1)$$

$$Z = H_1 H_2 H_3 H_4 S^V \quad (2)$$

where Q, b and V are constants.

Langley (1989) assessed a number of stochastic crack growth models, under constant amplitude loading and incorporating various density functions, against a set of experimental data. The results demonstrated the difficulties in the development of a crack growth model because although the models provide agreement for a particular set of experimental results they have poor predictive capabilities.

The sophistication of the R&R programs in implementing fracture mechanics differs. Crack growth in probabilistic programs may be incorporated by the implementation of one or more stochastic empirical crack growth equations, as described above. Some programs allow a tabular input of crack growth rates [Graham and Mallinson 1984] or

have an interfacing crack growth model or both. The computer program DARWIN includes a fracture mechanics module but may also interface to a user-supplied code [Leverant et al 1997]. DARWIN incorporates a life scatter factor, which accounts for material scatter and fracture mechanics modelling errors. The life computed from the crack growth model is multiplied by the scatter factor, which is a random factor lognormally distributed, producing a stochastic life prediction [Millwater et al 1999].

The fatigue crack growth data for specific alloys may be included in the codes' databases. The computer programs CARES/Life [Powers et al 1992] and DARWIN [McClung et al 1998] include crack growth data at several temperature levels to include the temperature effect on engine components. The latter program also allows a choice between crack growth data at air or vacuum environments [Millwater et al 1999].

For aging aircraft, the environmental effect of corrosion reduces the structural life by increasing the rate of crack growth. Orisamolu and Luo (1997) state that environmental effects on aging aircraft should be included in crack initiation and crack propagation models incorporated in the reliability analysis. They describe a modification of the power and Paris laws, incorporating the coefficient for corrosion in order to determine the reliability of an aging aircraft structure in a corrosive environment. The probability of failure was determined against the coefficient of corrosion, showing a rapid decrease in survival as corrosion increased.

3.1.2 Probabilistic initiation life

The crack initiation phase accounts for the majority of scatter in fatigue life [Tryon and Cruse 1997]. In contrast to crack-propagation, the probabilistic prediction of crack initiation is limited to simple testing cases of constant amplitude loading. Clearly, advancements in understanding crack nucleation and small crack growth are necessary in order to develop probabilistic models for crack initiation.

One example of application of probabilistic methods in crack nucleation is that of Tryon and Cruse (1997). The scatter inherent in the early stages of fatigue life was investigated using mesomechanics, the theory relating the microstructural heterogeneity to the scatter in crack nucleation life. The Tanaka and Mura model used describes the number of cycles to crack nucleation N_n (Equation 3) as

$$N_n = f \left(\frac{1}{(\Delta\tau - 2k)^2 d} \right) \quad (3)$$

where $\Delta\tau$ is the resolved shear stress, k is the shear strength needed to be overcome for dislocation motion, and d is the grain diameter.

The grain size, local resolved shear stress and mean frictional strength are treated as random variables. The reliability of crack nucleation life was determined using a specified number of cycles as the probability of failure criterion. The shape of the crack nucleation life distribution compared well with that from limited experimental data.

This provides an example of how probabilistic mesomechanical material models can link the uncertainties in material microstructure to the scatter in fatigue life.

Xiaoming et. al, (1998), developed a reliability analysis method for fatigue crack-initiation and crack propagation lives of structural components subject to variable amplitude loading. The method used an extension of the local strain-life approach and rainflow cycle counting of damage parameters to predict multi-axial crack initiation life. The method correlates the known local elastic-plastic strain state to crack initiation life [Xiaoming et al 1998].

Cruse, Mahadevan and Tryon (1997) describe a probabilistic risk assessment (PRA) program for in-service Retirement For Cause adopted by FAA for gas turbine engine disks. The method has successfully implemented damage tolerance approaches to commercial engines and is continuously evolving to account for advancements in the area. PRA includes probabilistic models of crack initiation and crack propagation and Probability of Detection (POD) analysis. Two methods of probabilistic analysis of fatigue life were available for PRA. The first used Weibull and lognormal distributions to describe scatter in fatigue life.

The second approach, which according to the authors is becoming the most popular one, is the use of fracture mechanics as the empirical form of the life model. This model uses a single crack size described as an equivalent initial flaw size to describe crack initiation. Initial results of the PRA application to gas turbine engine disks proved the fracture mechanics model to be non-conservative, as a consequence of multi-site cracking in the disks. Consequently the PRA model was revised to incorporate some of the multi-site damage effects [Cruse et al 1997].

A group of researchers developing a probabilistic design life code for turbine rotors is conducting tests to determine the time to crack initiation in samples of rotor grade Ti-6-4 material with hard alpha defects [Leverant et al 1997]. Initiation life models will be developed as a function of important defect characteristics and implemented in the probabilistic design life model. This will require an appreciation of the fundamental mechanism of crack initiation in the presence of hard alpha defects.

3.2 Computational Methods

The basic computation in determining the probability of failure, is the multiple integral of the probability density function in the region of failure (equation 4) [Moore et al 1990, Cruse et al 1997, Robinson 1998, Szekely 1998, Wu and Chen 1999].

$$P_f = \int_D \int f(x_1 \dots x_n) dx_1 \dots dx_n \quad (4)$$

where $D = \{(x_1 \dots x_n) : g(x_1 \dots x_n) < 0\}$ is the region of failure, P_f is the probability of failure, $f(x_1 \dots x_n)$ is the joint probability density function of the drivers $x_1 \dots x_n$, and the function g is the limit state which is the boundary between region of safety and region of failure. The drivers for fatigue failure may be the component strength and the applied load or the actual flaw size and the critical crack size [Alawi 1990] or the

operating time and fatigue life. The numerical evaluation of the above integral is difficult and time consuming and limited to simple cases.

A number of computational methods are used to carry out the probability computations required in risk or failure analysis including Monte Carlo simulation, First and Second Order Reliability Methods (FORM/SORM), Mean Value First Order (MVFO) and Response Surface Method (RSM). These employ either random sampling or analytical techniques. A comprehensive description of the methods is beyond the scope of this review; an excellent reference identifying strengths and weakness of each analytical method can be found in [Robinson 1988].

According to Cruse, Mahadevan and Tryon, (1997), limit state modelling approaches are gaining ground in estimating aircraft engine component and system reliability over traditional methods. The most significant contribution of limit state modelling is the computation of the sensitivities of the structural reliability to the basic random parameters.

Monte Carlo simulation has been adopted in probabilistic analysis because it can be used with failure modes of any complexity and can give high accuracy. It is a method used in many of the programs for aircraft and aerospace risk assessment [Ohnabe et al 1985, Leverant et al 1997]. The drawbacks to this approach are high computational time and the dependence of the statistical certainty of the estimates on the size of the sample [Szekely et al 1998, Xiaoming et al 1998].

The computational effort associated with Monte Carlo simulation can be reduced by limiting the number of simulation trials when the probability of failure is extremely low [Moore et al 1990]. The number of simulations normally necessary for accuracy of component reliability is in the order of 10^4 to 10^6 [Moore et al 1990, Lust and Wu 1998, Wu et al 2000]. A hybrid Monte Carlo method that is gaining interest is the Importance Sampling Method. This method increases computational efficiency because it samples random variable realisations in the failure region [SRI 1999, Wu et al 2000].

First and Second Order Reliability Methods (FORM/SORM) can transform the integral of equation 4 to an approximately equivalent one that can be efficiently evaluated. A description of the transformation process can be found in references [Moore et al 1990, Cruse et al 1997, Robinson 1998, Xiaoming et al 1998]. The transformation changes the complex integration problem into a mathematical optimisation problem, where the limit state function is approximated by a simply defined surface, linear (FORM) or quadratic (SORM). The Mean Value First Order methods (MVFO) use a linear approximation for the g function by expanding the limit state function about the mean value of the basic variables, thus not requiring formal optimisation as FORM/SORM. The critical step in these methods is the 'most probable point' search, for which many methods have been developed [Xiaoming et al 1998].

There are conflicting views on the efficiency of the computational methods. The FORM/SORM approach has not been considered appropriate for calculating probability of failure because for complex non-analytical limit state functions the problem becomes unstable [Moore et al 1990]. Monte Carlo simulation is preferred to FORM/SORM and MVFO methods due to higher accuracy of results [Moore et al

1990]. Other researchers describe FORM and SORM as much more efficient than Monte Carlo and reasonably accurate [Xiaoming et al 1998]. One source states agreement between Monte Carlo and Advanced Mean Value algorithm results when used to evaluate an explicit function [Lust and Wu 1998].

Szetely et al (1998) describe the use of the Response Surface Method (RSM) to fit an interpolation function through points of the limit state function of a complex structure, when the explicit function cannot be derived. The drawback is in not being able to quantify the error without knowing the real limit state function. The authors state that the computational effort can reach that for Monte Carlo simulations for a large number of random variables. As a result, this method is considered a promising tool for reliability analysis only when a limited number of random variables are present.

From the above it is shown that the computational methods used for determining the probability of failure differ in efficiency and the level of complexity. The Structural Safety and Reliability 8th International Conference in 2001, included a great number of papers on improved computational methods, with Monte Carlo and the Response Surface type methods receiving greater attention. It can be said that improvements in computational efficiency will be continuous, providing many options for future applications of probabilistic methods.

4. Data Requirements

4.1 Fatigue Life of Components

Fatigue crack growth data available in company databases or in the public domain show considerable scatter, which may introduce uncertainty in the component life prediction [Ohnabe et al 1985, Provan 1987, Min and Xiong 1992, AIARI Sub-Committee 1997, Tryon and Cruse 1997]. Fatigue life is affected by many parameters, not always measurable, such as thermal and residual stresses and environmental effects.

According to Melis & Zaretsky (1999), the effect of temperature gradients in a gas turbine engine disk is two-fold: -

- (i) Fatigue life of material generally decreases with increasing temperature, and
- (ii) Thermal gradients in the disk induce thermal stresses.

A mathematical expression is provided showing the thermal effect on disk relative life (equation 5), where ΔT is total radial temperature difference and K_T is a constant. If the thermal gradient is known the equation can be used to adjust the life prediction up or down. The authors note that besides the thermal gradients, the bulk temperature can affect fatigue life, but they admit there is lack of fatigue data showing the effect of elevated temperature.

$$L \approx \left[\frac{K_T}{\Delta T} \right]^{0.52} \quad (5)$$

Some fatigue crack growth programs include temperature interpolation of fatigue crack growth properties or the selection of properties at the nearest or next highest temperature in the database [Wu et al 2000].

It is similarly difficult to obtain information on compressive residual stresses present in disk material from heat treatment or shot peening. Compressive shear stresses can reduce effective shear stresses and thus increase life [Melis and Zaretsky 1999].

For aging aircraft, environmental effects, such as corrosion, can significantly reduce the life by increasing the crack growth rate of structures [Orisamolu and Luo 1997, Whaley 1998]. This was demonstrated by Orisamolu and Luo (1997) for airframe components in a study that showed corrosion to significantly reduce reliability levels [Orisamolu and Luo 1997]. The effect of corrosion on engine component reliability was not available in the literature reviewed.

G. Leverant et. al , (1997), discuss the effect the environment has on crack growth for titanium alloys, resulting in different crack growth rates for surface fatigue cracks and embedded cracks in the same material. This questions the relevance of fatigue life data obtained in air or vacuum testing conditions. The authors are conducting tests for titanium engine alloys in vacuum to complement the databases of data in air environments. The vacuum fatigue crack growth data is necessary for the risk assessment of components with sub-surface defects that effectively grow in a vacuum environment [McClung et al 1999].

An additional effect on the life of high-energy titanium engine rotors is that due to inherent material anomalies or induced during manufacturing or maintenance. Thus entering service and affecting the life of engine components. The presence of material anomalies in the microstructure of engine components necessitates the use of probabilistic methods to describe the stochastic nature of the effect [AIARI Sub-Committee 1997]. An FAA program is investigating the risk of failure from melt-related anomalies called hard alpha. These anomalies are hard brittle zones that have voids and cracks and are associated with low-cycle fatigue cracks that have lead to turbine disk failures. The quality of the component material prior to service is not perfect but characterised by flaws that influence the life of the component and must be accounted for in the R&R code.

The uncertainty in material behaviour that results from microstructural non-uniformity and material property degradation from loading has been quantified in a probabilistic model, the Multi-Factor Interaction Equation (MFIE) [Shah et al 1992, Jeremic and Bengin 1998]. The model assumes that material behaviour can be simulated by a number of primitive variables and describes how the material property M_p is degraded as the number of cycles increases (equation 6). The change in material property is expressed by the product of any number of effects. Each primitive variable V and exponent a , can be random and experimental data with engineering judgement are needed to determine them.

$$\frac{M_p}{M_{p_o}} = \prod_{i=1}^N \left[\frac{(V_f - V)}{(V_f - V_o)} \right]^a \quad (6)$$

where subscripts o, f refer to the condition at reference stage and final stage respectively. Examples of use of this model in R&R programs are given in [Jeremic and Shah et al 1992, Bengin 1998].

As well as the processing and environmental effects, post-production treatment of engine components can alter their fatigue behaviour. The fatigue life of gas turbine engine components can be significantly affected by machining operations, where inappropriate or abusive machining can increase crack growth rate and reduce the useful life [Cruse et al 1997, Tong and Kappas 2002].

As Cole (1998) points out, the accuracy of computations arising from the limiting assumptions in the reliability models should not hamper their use. He believes that models should be made as realistic as possible and should be calibrated against experience. He continues stating the difficulties associated with service data gathering:

- The process requires long-term commitment,
- Lack of precise definition of quantity to be measured,
- Existing data is censored,
- The critical component may not be detected,
- Cost may not be the driving force.

4.2 Availability of Data

"Risk estimates are commensurate with the amount of information input to the failure or probabilistic model" [Moore et al 1990].

General agreement exists among researchers as to the limitations in quality and volume of experimental data available to support reliability analysis of aircraft engines. Data is required as input to the probabilistic models and also for validation of the results.

For gas turbine disks, spin pit testing provides the means of determining crack growth characteristics and the life of engine components. The importance of spin pit testing of gas turbine engine disks is discussed in [Mahorter et al 1985b] and its use in probabilistic analysis has been described in [Ohnabe et al 1985, Melis and Zaretsky 1999].

The lack of fatigue crack growth rate data for titanium alloys in a vacuum and the effect of temperature and stress ratio on the fatigue crack growth rates are discussed in [McClung et al 1998]. This data was necessary to develop a probabilistic damage tolerance method for turbine rotors that are characterised by sub-surface hard alpha inclusions. Due to the lack of data, the researchers have conducted their own experiments on titanium alloys for fatigue crack growth in vacuum at different

temperature and stress ratios and have made the data available to the public [McClung et al 1998].

Besides the material and crack propagation characteristics of engine components, the stresses or operational loading is required and may be obtained from the mission and/or finite element analysis. The importance of using updated mission profiles to accurately determine the engine operating stresses has been much emphasised [Tong and Kappas 2002]. Le and Peterson (1999) describe how fatigue reliability assessment of in-service engine components can be achieved when no loading information exists. Fatigue testing of components removed from service provided the additional data necessary in terms of the damage accumulation over a period of time. The method compares the fatigue damage resistance of tested and new components to indirectly obtain the fatigue damage accumulation due to cyclic loading of in-service components. This data can be used to calculate the fatigue life of in-service components for a specified reliability level. The model has been used on compressor disks by conducting fatigue testing of a large number of retired disks. The limitation of this method lies in the availability of enough retired components.

A common feature of probabilistic fracture mechanics is the introduction of stochastic exponents in crack growth rate equations. This approach has been used to incorporate material variability in the Paris-Erdogan and Forman equations [Min and Xiong 1992, Xiaoming et al 1998]. Obtaining the experimental data to describe statistically the stochastic nature of the exponents is a difficult task. Wu and Chen (1999) proposed a systematic approach to determining density distribution functions for the input parameters when data is limited. They recommend the use of a spectrum of candidate distribution models to provide a range of reliability. The range of values can assist in building confidence in the reliability result, as many input assumptions have been covered.

Round robin studies are an excellent source of material and crack growth databases for use in R&R programs and at the same time rare due to costs. One such recent study is that of AGARD on Ti-6-4 forgings, which involved replicate tests at 13 laboratories and allowed the scatter in the experimental life to be assessed [McClung 1999]. Other programs at developing databases exist within engine OEM's but as was made clear in the PMC2001 conference, the data collected for example by United Technologies on a program on turbine blade thickness variability would not be provided to the public [Orisamolu and Luo 1997].

4.2.1 Benchmarking and comparing with statistical databases

Melis and Zaretsky (1999) state that the validation of any R&R model for engine components is difficult due to the limited amount of fatigue data. The limited amount of experimental data for use in probabilistic programs has been linked to the expense of full scale testing, the extensive time requirements and the difficulty in simulating all possible loading conditions [AIARI Sub-Committee 1997].

Majorter et al (1985) used low-cycle fatigue testing data for gas turbine compressor disks of titanium alloy (Ti-6Al-4V) generated at the Naval Air Development and the Naval Air Propulsion Centers, to compare with analytical life predictions made by the manufacturer. Five disks, each of two different types were LCF tested until a crack at the bolthole exceeded the 1/32" limit. Weibull analysis of results allowed the Weibull shape parameter and the corresponding life at one in a thousand failure to be predicted. The comparison of experimental and manufacturers analytical lives indicated that the analytical predictions were not consistently conservative.

Majorter et al (1985) discuss the issue of sample size for component testing and confidence levels of predictions stating that large sample sizes are not practically possible due to expense of spin pit testing. They discuss the loss of accuracy, due to the small sample size of five disks, as being a 20% error in the observed Weibull slope. Their analysis of the results at an 80% confidence level for various sample sizes shows a large variation in one in a thousand life predictions for the disks, due to the experimental scatter in values rather than the small sample size. The Weibull slope obtained (2.8-3.5) corresponded to a value between that obtained from materials test specimens and that from in-service parts performance. The results of the spin pit testing did not prove, as the authors expected, that the manufacturer's analytical predictions were conservative. On the contrary, the authors question whether engine components may be in operation beyond their one in a thousand lives. Based on their results, where they had difficulty in establishing the one in a thousand life, the authors recommend the use of a Retirement for Cause approach for gas turbine engine disks life management.

Melis and Zaretsky (1999) used the same LCF database to benchmark the predictions of the Probable Cause program. They state that the LCF data resulting from spin rig testing of disks was the only such failure data available in the open literature. They further comment that relevant data, including material processing and metallurgical characteristics of the titanium alloy, manufacturing process and engine operating temperature of each disk, were not available. As the authors mention, the unreported variables can affect the disk life and the results of statistical life analysis.

The obvious difficulty in validating life predictions shows how validation of R&R programs for engine components is also hindered.

4.3 Stochastic Drivers

According to Berens (1996) although there are many stochastic elements influencing the initiation and growth of cracks, most damage tolerance applications still use a deterministic approach. Moore, Ebbeler and Creager (1990) state that inputs to failure models described by intrinsic variability and specification error should be represented stochastically. They mention material properties and input loads as examples of intrinsic variation. They also describe specification error as originating from the accuracy of the engineering model and the limited or incomplete information regarding the physical parameters describing the failure process.

Many researchers treat the stochastic drivers as random variables of specified distributions. Cruse, Mahadevan and Tryon (1997) state that the random variables describing a reliability model should be physically measurable to determine the scatter. When a high number of potential random variables exist, the authors suggest performing sensitivity analysis to determine the significant random variables by response of the system. This method was used for a gas turbine engine system where the eleven initial significant random variables were reduced to five, by perturbation of the random variables in each of heat transfer, stress analysis and crack propagation models. The Paris law representation was then used to determine a performance function that more closely fitted the results of the fatigue life perturbation analysis.

4.3.1 Probability density distribution functions

Historically a probabilistic approach to determining lives of engine components was suggested by Palmgren in 1924 for rolling- element bearings and Weibull in 1939 for the scatter in material strength of solids. Weibull analysis has consequently been used to describe life of engine components.

Cole (1998) states that the Weibull distribution *"has been the most useful one for analysing aero-engine failures"*, because it can handle all three parts of the bathtub curve. The Weibull slope β , describes infant mortality ($\beta < 1$), random ($\beta = 1$) and wear-out ($\beta > 1$) sections of the bathtub curve. The author refers to the work by Dr R. Abernethy of Pratt & Whitney in early 1970s that lead to the development of the 'benchmark' working manual on Weibull analysis for jet engines. The manual includes techniques that deal with the censored nature of the failure data encountered in military applications.

It is often found in the literature that certain distribution functions are commonly used among researchers to describe variation in material properties. According to Provan (1987), a number of distributions can empirically describe the scatter in fatigue data, including exponential, lognormal, Weibull and Gumbell (extreme value). The author states that often the exponential distribution may be chosen as a failure model because it is easy to use rather than because of understanding the physical damage occurring in a heterogeneous material.

The Weibull and lognormal distributions have both been used to describe scatter in crack initiation life [Ohnabe et al 1985, Cruse et al 1997]. The mean value and standard deviation of the lognormal distribution for the time to crack initiation can be obtained from component testing and coupon testing respectively [Ohnabe et al 1985]. Weibull has been used to describe fatigue life [Mallinson 1982] and initial crack size for gas turbine engine disks [Cruse et al 1997]. Lognormal has been applied to describe variation in critical crack size [Ohnabe et al 1985] and the scatter in the stress and life predictions for gas turbine disks [Millwater and Wu 1992].

It is important though to ask whether the correct type of distribution function is used or previous choices simply followed. It has been stated that the probability of failure, P_f , is very sensitive to the choice of the type of input density functions used to describe the life or material properties [Wu and Chen 1999, Tong and Kappas 2001]. According

to Wu and Chen (1999), a drawback of probabilistic modelling is the need to accurately describe distributions for the input parameters. They state that the uncertainties from the models are minor compared to those from the input parameters, a fact which discourages some researchers away from the application of reliability models towards the use of the safety factor.

The choice of distribution functions has to be justified. A systematic approach to developing distribution functions for input parameters to reliability models when data is limited has been proposed by Wu and Chen (1999). The authors suggest reliability analysis be performed for a number of potential distribution functions describing the input random variables. A sensitivity analysis on the results will then identify the significant random variables to which confidence limits can be assigned and reliability bounds can be determined. The authors also discuss the significant effect of outliers (odd data points) on the distribution function describing the random variable, list methods for testing for outliers and state that the existence of outliers depends on the distribution assumptions. As stated in the conclusion of the reference, using a spectrum of distributions produces a range for reliability adding credibility to the reliability analysis result. The approach described by Wu and Chen (1999) was independently undertaken in the reliability study conducted in DSTO on engine turbine disks. The sensitivity analysis identified the significant input parameters for each of two of fan disk cracking incidents [Tong and Kappas 2001].

According to Moore, Ebbeler and Creager (1990), engineering analysis and past experience can be used to characterise the probability distribution functions when no extensive experimental measurements exist. In the case of limited information available, they state the importance of never overstating its significance. The following general guidelines for characterising stochastic drivers are provided:

1. Include all sources of driver uncertainty,
2. Driver information used should be traceable and documented,
3. The driver information should include available data, past experience and engineering analysis,
4. For physically bound drivers, the beta distribution has been used,
5. For drivers with scalar information only, the uniform distribution has been used,
6. Past experience assists in characterising drivers using particular distributions.

Often the lack of data hampers the accurate description of variability. One study describes how the mean and standard deviation of the critical crack size were determined from the values for mean and standard deviation of the fracture toughness [Whaley 1998]. Researchers using finite element analysis in combination with Weibull representation of material properties to determine the probability of survival of the turbine disk, comment that in the absence of known Weibull parameters it may be permissible to assume them [August and Zaretsky 1993].

Expert opinion can be utilised to describe the distribution of engine component life data, in the presence of limited experimental data using a PC-interactive approach [Singpurwalla 1988]. The underlying assumption in this approach is that the life data can be described by a Weibull distribution. Expert opinion is required to provide the

shape factor of the distribution and the median value and variability for the life. The paper describes two real life applications, one of sufficient life data and one of scarce life data. In the latter case failure data from spin-pit testing of five gas-turbine engine disks were complemented by expert opinion from OEM and the analyst to obtain a representative Weibull distribution of the life of the disk. The OEM expertise was used to deduce the median life and the variability for the disk. Comparison was made between the results of this approach with the analytical OEM lives. One important conclusion from the results was that the analytical OEM lives were inconsistently conservative. One may question the effect of subjective opinion of expert and analyst in this approach and emphasise the importance of understanding the nature of the failure mode prior to assigning a distribution function to limited data.

The variation in fracture toughness is required as input to many probabilistic models of fracture as representative of the variation in residual strength, or critical crack size [Graham and Mallinson 1984, UK DEFSTAN 00-971 1987, Whaley 1998]. The necessity of conducting experiments in order to justify the distribution chosen to represent fracture toughness of material, initial crack size, critical crack length is obvious. This need is crucial for obtaining crack information for aging structures suffering from corrosion. As Whaley (1998) states, the identification of the distribution describing the initial crack size for corroded material is a critical task, hampered by even less data than for non-corroded material. The author further states the need for experimental work to determine crack growth rate and fracture toughness of corroded material from aging aircraft structures. This may equally apply to engine alloys affected by corrosion.

According to Duffy, Powers and Starlinger (1993), the use of the three-parameter Weibull distribution has been limited due to lack of strength data and the difficulty in estimating the parameters. The authors propose a threshold stress to be used as the third parameter in the Weibull distribution such that zero probability of failure exists for applied stress at or below the threshold value. The application of the proposed method to a turbo pump blade using CARES/Life computer program showed better agreement with data and proved the two-parameter method to be conservative at all stress levels. Nevertheless, the authors carefully point out that further tests would be necessary to conclude whether a population behaves according to the two or three parameter Weibull distribution.

It is possible to incorporate test failure/success data in the development of a failure probability distribution [Moore et al 1990, Moore et al 1992b, Singpurwalla 1988]. A statistical approach using Bayes rule [Moore et al 1990] has been used to modify the *prior* distribution of a failure parameter by using success/failure data obtained from test or service experience. This Bayesian approach allows a prior failure probability distribution to be updated given the additional test failure data. It has been used to incorporate success/failure data in the reliability analysis for launch vehicle propulsion systems [Moore et al 1990], and for gas-turbine aircraft engine disks [Singpurwalla 1988]. It is emphasised that the existence of success/failure data for a component is an exception to most failure situations, as tests are rarely conducted to fail components. This comment certainly applies to launch propulsion systems, but also for military

engine disks, where spin pit tests can supply appropriate failure/success data but in a limited amount.

Inevitably testing has to be conducted to provide data for determining the correct distribution function. One program attempted to establish a statistically meaningful Weibull failure distribution for a limited number of engine disks by cyclic spin pit testing, to determine the life at one in a thousand failure and compare this with the analytical life prediction. Weibull was again chosen because it is the most commonly used distribution for aircraft design failures. The paper mentions a current investigation undertaken to determine the statistical distribution best suited for aircraft disk failures [Singpurwalla 1988]. Another study used a combination of component testing and coupon specimens to determine the lognormal distribution, assumed to best describe the time to crack initiation of jet engine disks. The mean value for the time to crack initiation was obtained from component test data under nominal loading spectra and the standard deviation was obtained from coupon specimens and statistical dispersion of service loads [Ohnabe et al 1985]. As mentioned at the PMC2001 conference, developing databases to define distributions is a current requirement in the aircraft manufacturing industry, but comes at a high cost [Rogers 2001].

The AIA Rotor Integrity Sub-Committee (1997) which has aimed to reduce uncontained gas turbine rotor events had difficulty obtaining the data needed for the development of a hard alpha anomaly distribution for engine risk assessment. A major difficulty in obtaining the data was that the available data corresponded to material samples with different shapes and therefore different inspectable cross-sectional areas that distorted the initial findings. An investigation of the fundamental mechanism causing the distribution of anomalies was necessary. A series of analytical models that simulated the manufacturing process, incorporating the observed data, was implemented to modify the database to that required for the baseline distribution. The final distribution was estimated from the adjustment of the baseline so that it matched that of commercial engine service experience over a period of years.

Clearly there has been limited effort to obtain an understanding of the fundamental mechanisms that cause variability in material properties and fatigue life. The assumption of a distribution function type and shape can produce inaccurate results and should be avoided. For risk assessment of engine components, the choice of probability density functions will be a difficult procedure [Tong and Kappas 2002].

4.4 Current Technology

Finite element analysis and non-destructive testing provide input to risk assessment programs. Any technological improvements in these fields will reflect on the development of R&R programs.

4.4.1 Finite element analysis

Finite element methods are incorporated in a number of reliability programs to determine stress distributions at a critical location or region [Moore et al 1990, Nemeth et al 1990, Shah et al 1992, Lust and Wu 1998, Hughes 2000].

R&R programs that are modular and have an interface with a finite element code include CARES/Life [Rahman et al 1998], NESSUS [SRI 1999], SPSLIFE [Saith et al 1995] and DARWIN [Wu et al 2000]. These go beyond the first level of only determining stresses and temperatures but also determine the probability of survival and the risk. Such programs determine the "risk" hot spots of an engine component.

August & Zaretsky (1993) describe a method that incorporates finite element analysis into component life and reliability adopting a discrete-stressed volume approach. The method uses shear stress results from FE analysis to determine the life and probability of survival for each computed element. The method has been used in the program Probable Cause where the survivability of each stressed volume as well as the survivability of the whole component can be determined, identifying spots for redesign. The method uses shear stress results from FE analysis to determine the life and probability of survival, S , for each element (Section 5.1).

In the DARWIN program, the stress in the component is treated as a random variable [Wu et al 2000]. The deterministic stress prediction from the finite element analysis is subjected to a multiplying factor that is a random variable. This random variable accounts for limitations in FE predictions and variability in geometric tolerances, which affect the result of the risk assessment. The stress-multiplying factor is assumed to have a lognormal distribution [Millwater and Wu 1992].

Szekely et al (1998) describes how stochastic modelling of a structure is achieved by treating the important parameters of a FE model as random variables and using Monte Carlo simulation. Commercial FE models can be utilised in the Monte Carlo simulation without significant changes in the internal structure of the FE analysis. The process that involves pre-processing, a loop of FE runs, and post-processing of the results has been found time-consuming and methods of reducing computational time are available [Hughes 2000].

The stochastic approach is the latest significant development in commercial FE programs. Probabilistic FE programs provide greater flexibility as the uncertainty in input parameters is incorporated. ANSYS and ProFES are two examples of software that determine component stress as a result of stochastic inputs. ANSYS has further expanded into stochastic crack growth analysis [Khor 2001]. ProFES allows probabilistic models to be developed from existing deterministic FE models and was developed initially for the turbine engine industry and with industrial partners including engine OEMs (PWA, GE and Rolls-Royce) [Cesare and Sues 1999].

4.4.2 Non destructive evaluation

The application of probabilistic methods for gas turbine disks described by Ohnabe et al (1985) demonstrated that improvement in fatigue reliability is sensitive to the crack detection capability of the NDE technique. NDE was applied to turbine engine disks to define the inspection procedure for Retirement For Cause of the disks. This was conducted at two stages to avoid erroneous removals. If the disk passed the first inspection, the disk was returned to service, while if the first inspection rejected the disk, a second inspection was undertaken before removing the disk from service. The industry is aware of the limitations of the NDE methods and has to assure there are no costs from unnecessary component removals.

Clearly, the NDE capability is extremely important in a Damage Tolerance or Retirement For Cause approach. Cruse, Mahadevan and Tryon (1997) state that the effectiveness of crack detection through NDE lies in the sensitivity of the method in terms of nominal detectable crack size and the reliability of detection. The latter is given in terms of the Probability Of Detection (POD) curves. As the authors state "*a key reliability question for NDE measures is, what is the largest crack that can be missed?*" The authors discuss that obtaining the true POD characteristics is the least developed part of a Retirement For Cause/Damage Tolerance approach, further clouded by the inaccurate setting of field inspection NDE limits on the basis of laboratory induced cracks.

NDE capability has been found inadequate in detecting multi-site cracking in gas turbine engines [Cruse et al 1997] and fatigue cracking due to pitting corrosion in aging aircraft [Harlow and Wei n.d]. Issues of accessibility and inspectability are well discussed in [Goranson 1997]. Reference [Harlow and Wei n.d] describes the difficulties in producing POD curves for corrosion-induced fatigue damage and the need to significantly reduce the detectable size.

Experience in the gas turbine industry has demonstrated that material anomalies may not be detectable with standard NDE, degrading the engine's structural integrity [AIARI Sub-Committee 1997]. Improvement in NDE capability would ensure defects with potential catastrophic consequences for fatigue life are detected earlier through inspection.

R&R programs may incorporate the NDE capability in the form of POD curves, developed from experimental data for a specific component structure. Safie and Hage (1991) describe a model that utilised data provided by Pratt and Whitney for engine material to derive the POD. The model describes the POD as a function of the length of the crack inspected and two crack growth parameters. However its application is limited to a population of structures that have the same material and geometrical characteristics.

The computer program DARWIN allows multiple PODs depending on the geometrical locations of the engine component and on whether the inspection is made at the start of operation or in the field [Leverant et al 1997]. This acknowledges the dependency of the NDE result on the geometrical location of inspection and the operational

procedures. One single POD cannot account for all locations, even on the same component. In DARWIN the POD curve is used for post-inspection definition of defect distribution and thus has significant influence on the risk assessment.

5. Applications

The programs available for risk assessment of engine components vary in terms of probabilistic methods, the number of random variables incorporated and the adaptability and flexibility to many modes of failure. The next generation of these programs is expected to adopt a modular structure that allows interfacing and is expandable to incorporate other failure modes or stochastic parameters. These programs will inevitably grow in their capability in order to be applicable to a wide range of engine components with different failure mechanisms.

A number of programs have been developed specifically for R&R analysis of aircraft engine components (Probable Cause, DARWIN, Probabilistic Risk Assessment, Cares/Life). There is a larger group developed for application in other propulsion systems such as for aerospace components (NESSUS), as well as programs for generic use. The following section reviews some of the programs with engine applicability and provides examples of applications.

5.1 Gas Turbine Engines

Design Assessment of Reliability With INspection (DARWIN) is a probabilistic damage tolerance computer code developed by Southwest Research Institute. The program was initiated by the FAA for gas turbine engine rotors following the Sioux City rotor failure [AIARI Sub-Committee 1997, Leverant et al 1997, McClung et al 1999]. It was developed in collaboration with four major US engine manufacturers, Honeywell, Pratt & Whitney, General Electric and Rollce Royce-Allison. The use of the DARWIN program does not replace the safe-life design methods but aims to provide a tool for engine manufacturers for risk management. The code focuses on low-cycle fatigue as a consequence of hard alpha defects in titanium disks but there are plans to expand it to include other alloys and other low-cycle fatigue failure modes such as those induced by porosity or abusive machining (Section 5.3).

DARWIN has finite element and graphical user interfaces. The fatigue crack growth module allows a choice of crack growth models and solutions including stress ratio and temperature effects and user-defined databases [McClung 1999]. DARWIN uses two random variables, the Stress Multiplying and Life Scatter factors, to describe the scatter in the finite element predicted disk stress and crack growth predicted disk life [Millwater and Wu 1992]. Thus the stress and the life of the disk are obtained as the product of predicted deterministic values and a random variable, (equations 7, 8). The temperature interpolation of alloy material properties is a favourable feature of DARWIN because temperature effects are significant for gas turbine operation. Monte

Carlo and Importance Sampling Method simulations are implemented [Leverant et al 1997].

$$\text{Stress} = X_1 * \text{FE Predicted Stress} \quad (7)$$

$$\text{Life} = X_2 * \text{Predicted Life} \quad (8)$$

The risk assessment in DARWIN is conducted on a zone basis, each zone consisting of elements with similar material and stress characteristics, but different defect occurrence rates. The zone-based approach allows the user to concentrate on specific locations of interest, allowing faster analysis. The probability of failure of a zone, P_i , is conditional to the presence of a material anomaly in the zone and is given by equation 9, where α_i is the rate of defect occurrence in the zone and P_f is the probability of fracture of the zone, with the anomaly. Both α_i and P_f are obtained from a distribution of material anomalies in the zone and from fracture mechanics respectively.

$$P_i = \alpha_i * P_f \quad (9)$$

The component probability of failure is obtained from the summation of the probabilities of failure from each zone (Section 6.1) and single defect occurrence is assumed for each zone. Future extensions include the presence of multiple defects in zones [Wu et al 1999].

References [Millwater and Wu 1992, Wu et al 2000] provide examples of DARWIN applications to industry-supplied models, showing comparison of computational methods used for refining the program capabilities. The results show the decline in risk of fracture for a population of disks when inspection maintenance is introduced. There is an expected increase of applications of DARWIN in gas turbine engines as this program has received a positive response from manufacturers and researchers. DARWIN applications to aircraft and helicopter components were presented at the 2001 Annual FAA/Air Force/NASA/Navy Workshop on the Application of Probabilistic Methods (PM) to Gas Turbine Engines.

Another probabilistic computer code is Probable Cause, described by Melis and Zaretsky (1999), based on the computational method of August and Zaretsky (1993) using a Weibull based methodology of a discrete stressed volume approach. Briefly, the method establishes a reference life, L_{ref} , a reference volume, V_{ref} , and a reference stress, S_{ref} , and uses shear stress, τ , results from finite element analysis to determine the life, L , and probability of survival, S , of each computed element, using equations 10, 11, where the stress-life exponent c and Weibull slope e are material specific. The authors used the element with the highest stress as the reference point.

$$L = L_{ref} \left[\frac{\tau_{ref}}{\tau} \right]^c \left[\frac{V_{ref}}{V} \right]^{1/e} \quad (10)$$

$$S = S_{ref}^{(L_{ref}/L)^c} \quad (11)$$

The life and probability of survival can be obtained for each mesh element. The probability of survival S , for the entire engine component model is obtained from multiplying the individual survivabilities (equation 12).

$$S = \prod_n^{i=1} S_i \quad (12)$$

An example of the application of Probable Cause to gas turbine engine disks is described in [Melis and Zaretsky 1999]. Finite element analysis predicted the shear stress distribution for the disks. A Weibull slope of two was chosen for the failure distribution requirement. The analysis showed the bolthole location to have the highest probability of failure, in agreement with experimental results. The reference includes values of input parameters and results, rarely found in similar papers. Calculations for the material-life factor for Ti-6Al-4V from experimental results have been included. The effect of thermal stresses and residual stresses on disk life was discussed but was not included in the analysis due to lack of data.

Many computer programs use Monte Carlo simulation for risk assessment, see section 3.2. Ohnabe, Funatogawa and Yang (1985) describe the application of Monte Carlo simulation for probabilistic fatigue analysis of gas turbine engine disks under scheduled inspection maintenance for Retirement For Cause. The method applied to TF-33 jet engine disks, used a stochastic crack growth rate model and implemented crack growth data from spin pit testing. A Monte Carlo simulation of 5,000 runs showed the reliability of the engine components to increase with the introduction of scheduled inspection maintenance.

Both FORM and Monte Carlo computational techniques were implemented in one application of fatigue reliability analysis to gas turbine engine disks [Cruse et al 1997]. In this example FORM calculations were used in a sensitivity analysis that determined the significant random variables affecting the fatigue life of engine disks. This indicated which engine variables should be controlled in order to extend the disk lives. Monte Carlo simulation of 100,000 disks for a number of discrete time-dependent random parameters was also performed. The disk fatigue life calculated from the two techniques differed, and reasons justifying the results are given in the report, indicating once again the importance of the input assumptions for reliability models. The system risk was determined for a two-stage high-pressure turbine with fatigue failure at four locations. The assumption was that system failure occurs when any of the four locations fail. A different set of random variables affect the failure mode of each location and thus an individual performance function and FORM analysis was developed for each failure mode. The system failure was computed as the union of the individual failure modes and the sensitivity analysis indicated the random variable most influential to the system failure probability. This is a very good example of combining analytical engineering and probabilistic analysis in reliability of engine components.

During the last few years a large research effort has been undertaken in reliability analysis to determine the feasibility of utilising ceramic components in gas turbines because of their favourable elevated temperature properties [Duffy et al 1993a]. A large number of papers have focused on the applicability of R&R methods for ceramic gas turbine components [Nemeth et al 1990, Duffy et al 1993a, Duffy et al 1993b, Sturmer et al 1993, Thomas and Wetherhold 1993, Nemeth et al 1994, Saith et al 1995, Salem et al 1995]. Ceramic failure has been described as a probabilistic event due to the variation of inherent flaw size and the consequent large variation in strength [Powers et al 1992].

The Ceramics Analysis and Reliability Evaluation of Structures (CARES/Life) program for advanced fast fracture reliability has been used by a number of researchers to evaluate the reliability of ceramic components experiencing thermomechanical loading [Nemeth et al 1990, Powers et al 1992, Duffy et al 1993a, Duffy et al 1993b, Sturmer et al 1993]. The program has been widely distributed to US research and government bodies. CARES/Life predicts the fast fracture reliability of laminated ceramic matrix composites, for a chosen flaw shape and fracture criterion. The program utilises a range of models including two-parameter Weibull distribution for component strength variation and Paris law, Power law or Walker law for subcritical crack growth. A number of methods can be used to model the effects of multi-axial stresses, including the principle of independent action, the Weibull normal stress averaging or Batdorf theory. The program includes a database of material properties at a variety of temperatures. Details of the models employed in this program can be found in [Nemeth et al 1990, Powers et al 1992, Nemeth et al 1994]. As mentioned by Nemeth et al (1994) there are many potential enhancements to this program including the assessment of reliability for creep and oxidation failure modes.

Cares/Life has been applied to gas turbine disks [DiMascio et al 1998] and metal-matrix composites [Zaretsky 1996]. One application to turbine disks showed the reliability predictions to be much more sensitive to the accuracy of the finite element model than to the strength of the material parameters [DiMascio et al 1998].

One important issue for the reliability analysis of ceramic matrix composites materials is the size effect in the fibre direction of a unidirectional material [Duffy et al 1993a]. The choice of a parallel or series system for the structure is not straightforward, especially when the material properties are believed to be dependent on the test method used. There is concern of lack of experimental data that would validate reliability programs for ceramics.

Another program that models sub-critical crack growth in ceramics is described in [Sturmer et al 1993]. The program determines the thermal and stress distributions and calculates load and time-dependent failure probabilities. One example of a gas turbine component showed the results using different fracture criteria to diverge with increasing time, emphasising the importance of selecting the correct fracture criterion.

SPSLIFE is another program used for the design assessment of ceramic components for fast fracture, slow crack growth, creep and oxidation. Results of the application of SPSLIFE to a ceramic gas turbine component were compared to results from

CARES/Life and good agreement was found between the two programs [Saith et al 1995].

5.2 Propulsion systems

The Numerical Evaluation of Stochastic Structures Under Stress (NESSUS) computer program has been developed by NASA Lewis Research Centre to simulate response to uncertainties in structural parameters and to assess the reliability and risk for structural components of space propulsion systems. It provides component and system risk assessment and claims to be "*the first major probabilistic finite element program that could solve complicated reliability analysis problems such as a turbine blade with multiple failure modes*" [Millwater and Wu 1992, SRI 1999]. An extensive list of NESSUS-related publications is found in [SRI 1999].

This is a multi-functional and modular program, integrating various probabilistic and finite element methods and utilises a number of probabilistic methods including Advanced Mean Value and Importance Sampling Method. It is a generic program and hence is applicable to any propulsion system component. The Multi-Factor Interaction Equation (MFIE) (Section 4.1) is used to simulate material property/strength degradation due to cyclic loading on the engine component.

NESSUS has been applied to a large range of aerospace structures including turbine blades and main combustion chamber, investigating the high cycle and low cycle fatigue response of these systems. A summary of the applications is given in [SRI 1999]. A number of applications investigate the effect of thermal loads on engine structures utilising a thermal analysis code. This latter feature sets NESSUS as a more advanced probabilistic tool compared to other R&R programs that cannot implement thermal effects in the probabilistic assessment.

NESSUS has been used in sensitivity analyses to determine the dominant variables affecting the reliability of the Space Shuttle Main Engine turbine blades [Shah et al 1992]. The study considered the centrifugal, thermal and pressure loads, material properties and geometry as random variables expressed by probability density functions. The application emphasised the need to determine the fundamental mechanisms that govern material behaviour to produce Multi-Factor Interaction Equations.

The Probabilistic Failure Assessment (PFA) methodology, described by Moore, Ebbeler and Creager (1990, 1992), uses Monte Carlo simulation to determine failure analysis for launch propulsion systems. The program uses a Bayesian statistical framework to combine information from engineering analysis with success/failure data and obtain a failure risk estimate, similar to the method described for gas turbine engine disks [Singpurwalla 1988].

One study of the reliability of a single-engine aircraft evaluated the Full Authority Digital Electronic Control system [Hjelmgren et al 1998]. The analysis looked at faults of electronic parts of engines, using a reliability requirement of one in a million failures

per flight hour as specified in the US military specification MIL-F-9490. A comment was made of the lack of research papers on engine control systems compared to those for aircraft control systems. Some aspects of this study are of interest to engine applications, such as the description of the engine control systems as parallel or serial structures and the calculation of the probability of failure using the Markov chain method.

5.3 Inspection Maintenance

ADF interest goes beyond risk assessment towards the requisite management of risk. Moore, Ebbeler and Creager (1992) state that there are three options available when probabilistic assessment shows the risk estimation of engine components to be unacceptable. It is possible to:

- (i) obtain additional information to improve the quality of the product (re-design),
- (ii) life limit the parts and/or
- (iii) increase the inspection frequency.

Re-design and life-limiting the parts are expensive options for the user and can be avoided with the introduction of inspection intervals. This can be implemented with the assistance of R&R programs.

The future of R&R programs appears to lie in the application of risk assessment to fleet risk management through in-service inspection planning. This is achieved under a retirement-for-cause or damage tolerance approach to component life management. Although this does not imply replacing the safe life method, it can be used to complement it, as recommended by the FAA for turbine components following the Sioux City accident [AIARI Sub-Committee 1997].

Models that include inspection planning have been applied to a number of failure modes, including high cycle and low cycle fatigue and stress rupture of propulsion systems [Moore et al 1990, Safie and Hage 1991, Wu et al 2000]. These models recommend inspection of components at a specified interval and if a certain criterion is met their removal. Inspection of the components is conducted at an interval corresponding either to a critical crack size [Safie and Hage 1991] or a specified risk limit [Tong and Kappas 2001]. The removal of components can be based on a POD curve implemented in these programs [Wu et al 2000].

Ohnabe, Funatogawa and Yang (1985), describe one example of application of Monte Carlo simulation for TF-33 jet engine disks under scheduled inspection maintenance. During each maintenance inspection, the component was replaced by a new one if the crack size in the component was greater than a specified crack size. The authors state that when scheduled inspection maintenance was simulated for a disk that had a design life of 2,500 cycles, the disk service life was extended to 12,500 cycles. The results of the simulation showed, that the probability of failure decreased and the average percentage of replacement increased as the number of inspections increased. It was demonstrated that the introduction of scheduled inspection maintenance improves

the reliability of engine disks in service. Similar results were described for the assessment of turbine wheels, where reliability was increased at a lower rejection flaw size and decreased at longer inspection intervals [Safie and Hage 1991].

The computer program DARWIN calculates the risk of disk fracture due to defects without inspection by numerical integration, and then determines the risk of disk fracture with inspection (equation 13). Following the inspection and removal of disks, the program determines the new probability density function (PDF) of defect occurrence (equation 14). Application to industrial cases showed the probability of failure decreased with inspection maintenance [Wu et al 1999]. DARWIN treats the inspection time as a random variable described by a normal distribution.

$$P_f(\text{inspection}) = P_f(\text{no inspection}) * \frac{\text{No. of failures with inspection}}{\text{Total number of samples}} \quad (13)$$

$$\text{Post inspection PDF} = \text{Pre-inspection PDF} * \text{Probability of Non Detection} \quad (14)$$

The PRObability Of Fracture (PROF) program has been applied to aging aircraft to determine the inspection intervals that corresponded to acceptable risk of failure levels [Berens 1996]. A cost analysis of maintenance costs model is included and applications have suggested a cost benefit when shorter inspection intervals are applied than those determined from deterministic damage tolerance analysis. PROF has been applied to airframe structural components, where an inspection maintenance approach is widely accepted. A similar approach for engine components could be justified if the cost benefit can be demonstrated.

5.4 Sensitivity Analysis

Researchers often state that a sensitivity analysis in reliability studies can identify which parameters affect the system response the most, subsequently removing the non-important ones from the probabilistic analysis or concentrating effort on the most influential [Cruse et al 1997, Jeremic and Bengin 1998, Robinson 1998]. The relative influence of the random parameters is component and case dependent. According to Moore et al (1992), a probabilistic risk sensitivity analysis can detect sources of unacceptable failure risk requiring corrective action. The latter can be achieved through design revision, improvement in knowledge of the load, manufacturing process, environment and material behaviour or in the analytical models.

A sensitivity analysis of the relative importance of the random variables used in fatigue reliability is described in Cruse et al (1997). The method determined which parameters were most influential and therefore should be controlled in order to increase the fatigue life of gas turbine engine disks. The random variables included life, mission and engine parameters. It was concluded that one particular engine parameter, the burner outlet temperature had greater influence on the gas turbine disk life. Such an investigation can assist in identifying the parameters affecting disk life which may be regarded as deterministic in the risk assessment and identify which ones would have to be represented by distribution functions.

The relative effect of a number of random variables on the prediction of a corrosion-enhanced, fatigue crack growth model was investigated in a sensitivity analysis described in [Orisamolu and Luo 1997]. The results showed that the reliability reduction due to material loss from corrosion in aging aircraft is more significant than that due to corrosion enhanced crack growth rates.

A sensitivity analysis using the program DARWIN determined the relevant influence of the variability in each of the Life Scatter factor, Stress Multiplying factor, defect size and disk inspection time on the failure probability prediction [Enright and Wu 2000]. The defect size was shown to have the dominant influence on the lifetime reliability of the turbine rotor disks. The results were specific to an idealised rotor disk and are expected to differ for an in-service component.

The use of a spectrum of distribution functions allows the sensitivity of the risk prediction to the input variables to be determined. This has been recommended as a method that may increase the confidence in reliability methods by providing a range in risk output when all possible input assumptions are covered [Wu and Chen 1999].

As C. Chamis, Senior Technologist from NASA Glenn Research Lab, stated at the PMC2001 [Chamis and Abumeri 2001], the costly experiments needed for defining probability density functions can and should be avoided by quantifying a model's response to a number of different distribution function inputs. Sensitivity analysis thus identifies the most significant system parameters for a subsequent testing programme.

6. Further research

6.1 Engine Component and System Analysis

The risk of failure of an entire engine system cannot be easily determined. An engine consists of a number of critical components that may have more than one failure mode. The risk of failure at system level cannot be easily quantified and limited research has been conducted for engine system reliability. Some programs previously described treat a single component as a system of elements and determine the risk at the elemental level.

Cole (1998) describes how notional reliability targets were obtained in the Failure Modes, Effects and Criticality Analysis (FMECA) for an engine system. An estimate of the system level failure rate was obtained from a simple addition of the failure rates of each component of the system. In this traditional approach, each failure rate is simply obtained by dividing the number of component failures by the total operating time. This approach assumes the availability of an extensive database of failure history of both system and components. It also implies that previous component history can be of benefit in determining the average failure rate for the system. Such an approach

does not take into consideration the changes in operating conditions, which would have significant influence on the failure response of the system.

As mentioned in Shah et al (1992), different models for integrating component performance with failure uncertainties need to be developed, in order to evaluate the overall system reliability. Such models must account for multiple failure modes, interactions between different components and must evaluate the effect of individual component performance on the overall system. NESSUS is one such program that is being expanded to incorporate engine system risk assessment [SRI 1999].

The system reliability problem is well discussed in Cruse et al (1997), with particular reference to the issue of multiple crack initiation in gas turbine engine disks. The engine is described as a system of parallel or series networks. A series or weakest-link system is one in which the violation of any one of the design limit states causes system failure and the probability of system failure is computed through the union of the individual failure events. A parallel /redundant or fail safe system fails only when all the individual modes are violated and the probability of system failure is the probability of joint occurrence of all the individual failure events.

However, there are difficulties associated with the joint probability calculation when non-linear limit states exist. The existence of several modes of failure and their influence on each other is another difficulty in calculating the system reliability of gas turbine engine components. Cruse et al (1997) present a study of a high-pressure turbine system where it was assumed that system failure occurred when any of four locations failed. Reliability analysis was performed on each of the four individual failure modes and the overall system probability of failure was computed from the union of the individual reliability results.

The survivability of an engine component is often determined at the elemental level first, as with the programs Probable Cause [Melis and Zaretsky 1999] and CARES/Life, or at the zone level, as with the program DARWIN [McClung et al 1999]. Once the survivability of each element or zone is determined, the probability of survival for the entire engine component, S , (Equation 15) is then obtained as the product of the individual survivabilities, assuming a series reliability system. The series reliability model accounts for the failure of the engine component when any of the locations defined by the element or zone in the component fails.

$$S = \prod_{i=1}^n S_i \quad (15)$$

DARWIN determines the probability of failure at each zone in the component, P_i , and then obtains the sum for the component probability of failure, using equation 16. This assumes that failure in each zone is independent of other zones and the individual probabilities are small, (Appendix 1 provides mathematical explanation) [Leverant et al 1997].

$$P_f = \sum_{i=1}^{i=n} P_i \quad (16)$$

Competing Risk is a program that can be used to quantify the risk of failure of commercial engines due to multiple failure modes. Weibull analysis is used to determine individual Weibull lines for the failure rates of each in-service problem [Cole 1998]. The method determines the system risk as the product of the risks due to different failure modes. The author cites an example showing how the system failure rate can be produced for an engine with multiple failure modes. According to the author, the method used for commercial engines cannot be applied to military gas turbine engines because of higher component complexity and limited data availability.

Melis and Zaretsky (1999) discuss the reliability and lifing issues of a component with multiple failure locations, such as a disk with many boltholes. The disk life for one in a thousand failure of a disk with multiple boltholes can be calculated from the analytical prediction of a disk with a single bolthole. The life is calculated depending on the number of boltholes n and the Weibull slope β (Equation 17). The effect of a multiple number of boltholes is to decrease the life at which one in a thousand disk failures occur. However, there is no evidence in the literature of the OEM actually using this method to determine the safe life of engine disks.

$$L_{sys} = \frac{L}{n^{1/\beta}} \quad (17)$$

The above equation may have application in the case of multi crack initiation, with n being the number of sites with crack initiation in the same disk. Similarly, it may be used to describe any series reliability system, i.e. the effect on total engine life when a number of identical critical components are present. The development of system risk models for military engine systems will need to consider the lack of substantial databases and the complexity of jet engine systems. Further work has been conducted in DSTO as a result of this review, in the determination of the engine system risk when multiple critical components or multiple failure modes exist, and the application of the DEFSTAN 00-971 risk criteria [Kappas and Antoniou, 2002].

6.2 OEM Collaboration

It is encouraging to the aircraft and aerospace community to see the involvement of aircraft and aerospace engine manufacturers in research projects that aim to quantify the risk and reliability of engine structures. This sets a precedent for future directions.

The FAA sponsored software program DARWIN, the product of a four year FAA research, engineering and development grant with SwRI, was developed in collaboration with engine manufacturers Honeywell, Rolls-Royce-Allison, General Electric and Pratt & Whitney. The program has been granted a top 100 US Research and Development award for 2000 and is described as a major breakthrough in the FAA's safety research programs [Hughes 2000].

DARWIN has received strong acceptance by most turbine engine manufacturers worldwide. This is a positive step towards improving engine lifing methodologies by allowing risk assessment to complement the traditional safe life method. It acknowledges that traditional approaches to fatigue lifing of gas turbine engine disks

do not necessarily consider material anomalies that may have catastrophic consequences. This FAA sponsored code will have direct implications for commercial engines and it would obviously provide a precedent for military engines.

Probabilistic Structural Analysis Methods (PSAM) is the research and technology project undertaken by NASA-LeRC and is sponsored by the National Aeronautical and Space Administration [SRI 1999]. It is developed in order to increase system durability, assist in system certification of space propulsion systems, and has provided tools for low-cost engine design and development. The software code NESSUS resulted from this sponsorship and the collaborative efforts of SwRI and Rocketdyne. NESSUS was completed in 1995 and made available for component and system risk assessment of space propulsion systems. Rocketdyne had an active role in the project, validating the applicability of the code to the design of future engines.

The NESSUS technology has been applied to aerospace propulsion structures by Rocketdyne and by NASA, SwRI and many industries to other structural systems including aircraft, automotive and offshore structures. The NESSUS code has stimulated other risk code developments and the PSAM project has made valuable contributions to the open literature.

These two examples provide excellent precedents for the involvement of OEMs in the development of R&R tools and their subsequent application to aircraft and aerospace engine systems.

One further example is the *Disk Optimisation Program*, initiated in 1990 by the U.S. Air Force Research Laboratory and in collaboration with engine OEMs (Pratt & Whitney and General Electric) to promote the probabilistic design of gas turbine engines, in order to reduce weight and improve durability. The objective was to incorporate guidelines for application of probabilistic methods to the design of gas turbine engines in the ENgines Structural Integrity Program (ENSIP) by 2006. An example of the cost effectiveness of the program was given at the PMC2001 conference, where the LCF life of a second stage fan disk extended to twice the limit life was quoted as saving US\$148 million [Brown 2001]. The use of the 'power by the hour' instead of the engine ownership approach was noted in the same conference as a change that will promote the use of probabilistic methods in the design of engine components. This is because probabilistic methods are seen by OEMs as a tool that can help to reduce the cost of operating engines.

7. Concluding remarks

There are several current activities in the aerospace industry that aim to reduce the current gap between the risk assumptions of the safe life methodology, the military standards and the operational requirements for aircraft gas turbine components. These include the FAA Titanium Rotating Components Review program, the U.S. Air Force

Blade Optimisation program and the annual FAA/Air Force/NASA/Navy workshops on the application of probabilistic methods to gas turbine engines. The aim of these collective activities is to increase the capability of the aerospace industry in probabilistic methods for the risk and reliability assessment of critical engine components.

This report has reviewed the issues that drive the change away from a deterministic towards a stochastic approach in the design and analysis of engine component lives. Alternative lifing approaches that incorporate risk and reliability methods into the management of military engine components have been examined.

The availability of computer programs and methods for the risk and reliability assessment of gas turbine engine components has been addressed. Of particular interest are the modular programs resulting from collaboration between research organisations and the OEMs. Some of these programs incorporate an inspection maintenance approach and thus have potential use when a Retirement for Cause or Damage Tolerance approach is adopted for engine components.

The dependence of risk and reliability assessment on the availability of engine data has been discussed. This is a fundamental issue for gas turbine engines where thermal, manufacturing and machining effects on material properties need to be taken into consideration in understanding the failure modes. For gas turbine engines with military applications, databases describing material properties of alloys are not widely available, adding further difficulties in determining the probability density functions necessary for accurate risk and reliability assessments.

The review has referred to the military standards in determining what is the acceptable risk of failure of gas turbine engine components. Although estimates of the acceptable risk are given in UK DEF STAN 00-971, other standards lack quantified risk limits.

It is evident that risk assessment of gas turbine engine components using probabilistic methods is attracting considerable interest and effort in the military aviation community and a move away from the use of deterministic methods towards the implementation of stochastic approaches in engine lifing is in progress.

The report has highlighted areas in need of further research, including the effects of multiple failure modes incorporating creep and oxidation effects on risk of failure for engine components, and the development of engine system risk models.

8. References

1. Aerospace Industries Association Rotor Integrity Sub-Committee, "The Development of Anomaly Distributions for Aircraft Engine Titanium Disc Alloys", 1997
2. H.Alawi, "Designing Reliably for Fatigue Crack Growth Under Random Loading", Engineering Fracture Mechanics Vol. 37, No. 1, pp. 75-85, 1990

3. R. August and E.V. Zaretsky, "Incorporating Finite Element Analysis into Component Life and Reliability", *Trans. ASME, J. Mechanical Design*, Vol.115, No.4, 1993, pp. 706-710.
4. Major T. Bartsch, "Engine Structural Integrity", *Flying Safety*, pp 24-27, May 1985
5. A.P. Berens, "Applications of Risk Analysis to Aging Military Aircraft", *31st International SAMPE Symposium and Exhibition: Aging Systems, Affordability, Alternative Applications*, Society for the Advancement of Material and Process Engineering (USA), pp.97-107, 1996
6. J.M. Brown, "AFRL Turbine Engine Division Efforts Towards Probabilistic Design", *PMC 2001 Conference*, Newport Beach, California, USA
7. M.A. Cesare and R.H.Sues, "ProFES Probabilistic Finite Element System-Bringing probabilistic Mechanics to the Desktop", *AIAA99-1607*, 1999
8. C. Chamis (NASA GRC) and G.H. Abumeri (Dynamics, Inc), "Probabilistic Substructuring for Assured Reliability Evaluations", *PMC 2001 Conference*, Newport Beach, California, USA
9. D.D. Cioclov, "A Model for Fracture Risk Analysis" *Rev.Roum. Sci. Tech. Appl.*, Vol. 31, No.6, Nov-Dec 1986, pp 613-622
10. G.K. Cole, "Practical Issues Relating to Statistical Failure Analysis of Aero Gas Turbines", *Proceedings of the Institution of Mechanical Engineers; Part G, Journal of Aerospace Engineering*, Vol. 212, No. 3, 1998, p167-176
11. T.A. Cruse, S. Mahadevan and R.G. Tryon, "Fatigue Reliability of Gas Turbine Engine Structures" *NASA/CR-97-206215*, Lewis Research Centre, 1997
12. P.S. DiMascio, R.M. Orenstein and H. Rajiyah, "Reliability of a Conceptual Ceramic Gas Turbine Component Subjected to Static and Transient Thermomechanical Loading", *Journal of Engineering for Gas Turbines and Power*, Vol. 120, April 1998, pp 263-270
13. S.F. Duffy, J.L. Palko and J.P. Gyekenyesi, "Structural Reliability Analysis of Laminated CMC Components", *Journal of Engineering for Gas Turbines and Power*, Vol. 115, January 1993, pp 103-108
14. S.F. Duffy, L.M. Powers and A. Starlinger, "Reliability Analysis of Structural Ceramic Components Using a Three-Parameter Weibull Distribution", *Journal of Engineering for Gas Turbines and Power*, Vol. 115, January 1993, pp 109-116
15. M.P. Enright and Y.T. Wu, "Probabilistic Fatigue Life Sensitivity Analysis of Titanium Rotors", *Proceedings, 41st Structures, Structural Dynamics, and Materials Conference*, Atlanta, GA, April 3-6, 2000
16. B. Goldberg and V. Verderaiame, "Quasi-Static Probabilistic Structural Analyses Process and Criteria", *NASA/TP-1999-209038*, 1999

17. U.G. Goranson, Boeing Commercial Airplane Group, "Fatigue Issues in Aircraft maintenance and Repairs", *International Journal of Fatigue* Vol. 20, No. 6, pp. 413-431, 1997
18. A.D. Graham and G.D. Mallinson, "NERF-A Computer Program for the Numerical Evaluation of Reliability Functions-User Manual", DSTO, Structures Report 411, 1984
19. D.G. Harlow and R.P. Wei, "Probabilities of Occurrence and Detection of Damage in Airframe Materials",
20. K. Hjelmgren, S. Svensson and O. Hannius, "Reliability Analysis of a Single-Engine Aircraft FADEC", *Proceedings Annual Reliability and Maintainability Symposium*, 1998
21. W.J. Hughes, "R&D-sponsored Technology Wins Industry Award", *Intercom, Technical Center*, Vol 3, Issue 7, July 2000
22. S. Jeremic and A. Bengin, "Reliability Evaluation of Aerospace Components", 21st ICAS Congress Melbourne Australia, ICAS Paper 98-7,9,3, 1998
23. J. Kappas and R.A. Antoniou, "Engine System Risk and DEFSTAN 00-971", DSTO-TR, to be completed in 2002
24. S. Khor (ANSYS, Inc), "Probabilistic Design - Bringing Engineering Closer to Reality", PMC 2001 Conference, Newport Beach, California, USA
25. A.K. Koul, R. Thambouraj, M.D. Raizenne, W. Wallace and M.C. DeMalherbe, *Practical Experience with Damage Tolerance Based Life Extension of Turbine Engine Components*, NAE-NRC report, 1985
26. R.S. Langley, "Stochastic Models of Fatigue Crack Growth", *Engineering Fracture Mechanics*, 1989
27. X. Le and M.L. Peterson, "A Method for Fatigue Based Reliability When the Loading of a Component is Unknown", *International Journal of Fatigue* Vol. 21 pp 603-610, 1999
28. Lepikhin, V. Moskvichev, S. Doronin; "Statistical Fracture Modeling of Weld Joint for Nuclear Reactor Components" *Theoretical and applied fracture Mechanics* 29 (1998) 103-107
29. G.R. Leverant, D.L. Littlefield, R.C. McClung, H.R. Millwater and J.Y. Wu, "A Probabilistic Approach to Aircraft Turbine Rotor Material Design, Paper 97-GT-22, *ASME International Gas Turbine & Aeroengine Congress*, 1997
30. Y. Liu, S. Xi'an and L.Wang, "Fatigue Reliability Analysis for Flexible Nozzle of Solid Rocket Motor", 35th AIAA/ASME/SAE/ASEE Joint Propulsion Conference, AIAA 99-2651, 1999
31. R.V. Lust and Y.T.J. Wu, "Probabilistic Structural Analysis-An introduction", *Experimental Techniques*, September/October, 1998

32. R. Mahorter, S. Fowler and J. Salvino, "Statistical Analysis of Spin Pit Failure Data to Predict Inservice B.1 Lives of Gas Turbine Discs", Naval Air Development Centre, 1985
33. R. Mahorter, G. London, S. Fowler and J. Salvino, "Life Prediction Methodology for Aircraft Gas Turbine Engine Discs" in Proceedings of the AIAA/SAE/ASME/ASEE 21st Joint Propulsion Conference, New York: American Institute of Aeronautics and Astronautics, pp 1-6, 1985
34. G.D. Mallinson, "On the Genesis of Reliability Models", DSTO, Structures Report 393, 1982
35. R.C. McClung, B.H. Lawless, M. Gorelik, C. Date, Y. Gill, R.S. Piascik, "Fatigue Crack Growth of Titanium Rotor Alloys in Vacuum and Air" *Proceedings Symposium Fatigue Behaviour Titanium Alloys*, 1998
36. R.C. McClung, G.R. Leverant, Y.T. Wu, H.R. Millwater, G.G. Chell, C.J. Kulman, Y.D. Lee, D.S. Riha, S.R. Johns and P.C. McKeighan, "Development of a Probabilistic Design System for Gas Turbine Rotor Integrity", 7th International Fatigue Conference, Beijing, China, June 8-12, 1999
37. R.C. McClung, "Fracture Mechanics Analysis in DARWIN", 4th Annual FAA/USAF Workshop, Application of Probabilistic Methods to Gas Turbine Engines, Dec 7-8, 1999
38. M.E. Melis, E.V. Zaretsky; "Probabilistic Analysis of Aircraft Gas Turbine Disc Life and Reliability", NASA/TM-1999-107436
39. H. Millwater, Y.T. Wu, "Structural Reliability Assessment (SRA) Capability in NESSUS" Southwest Research Institute, Advanced Earth to Orbit Propulsion Technology 1992, NASA Conference Publication No.3174, VolII, pp.511-521, 1992
40. H. Millwater, J. Wu, M. Enright, D. Riha, G. leverant, C. McClung, G. Chell, C. Kuhlman, S. Fitch, "A Probabilistically-based Damage Tolerance Analysis Computer Program for hard alpha Defects in Titanium Rotors", 1999 USAF Aircraft Structural Integrity Program Conference, 30 Nov-2 Dec, 1999, San Antonio
41. L. Min and Y.Q. Xiong, " A Probabilistic Model for Fatigue Crack Growth", *Engineering Fracture Mechanics*, Vol. 43, No.4, pp651-655, 1992
42. N.R. Moore, D.H. Ebbeler and M. Creager, "A Methodology for Probabilistic Prediction of Structural Failures of Launch Vehicle Propulsion Systems", Paper No. 90-1140-CP, *Proceedings of the AIAA 31st Annual Structures, Structural Dynamics and Materials Conference*, April 1990, pp 1092-1104
43. N.R. Moore, D.H. Ebbeler and M. Creager, "Probabilistic Service Life Assessment" AD-Vol. 28, *Reliability Technology*, ASME 1992

44. N.R. Moore, D.H. Ebbeler, S. Sutharshana and M. Creager, " Probabilistic failure Risk Assessment for Structural Fatigue", Jet Propulsion Laboratory, 1992
45. North Atlantic Treaty Organization, Advisory Group for Aerospace Research and Development, Structures and Materials Panel, Proceedings Meeting 60th *Damage tolerance concepts for critical engine components*, San Antonio Texas, 1985
46. N.N. Nemeth, J.M. Manderscheid and J.P. Gyekenyesi, "Design of Ceramic Components with the NASA/CARES Computer Program", Lewis Research Centre, NASA-TM-102369, 1990
47. N.N. Nemeth, L.M. Powers, L.A. Janosik and J.P. Gyekenyesi, "Durability Evaluation of Ceramic Components Using CARES/LIFE", NASA-TM-106475, 1994
48. H. Ohnabe, O. Funatogawa, J.N. Yang; "Monte Carlo Simulation for Probabilistic Fatigue Analysis of Jet Engine Discs", 4th *International conference on Structural Safety and Reliability*, 1985
49. I.R. Orisamolu and X. Luo, "Probabilistic Assessment of Corrosion Effects on the Damage Tolerance of Aircraft Structures", AIAA-97-1070-CP, 1997
50. L.M. Powers, A. Starlinger and J.P. Gyekenseyi, "Ceramic Component Reliability with the Restructured NASA/CARES Computer Program", NASA-TM-105856, 1992
51. J.W. Provan, Probabilistic Fracture Mechanics and Reliability, Martinus Nijhoff, Dordrecht, 1987
52. S. Rahman, N.N. Nemeth and J.P. Gyekenyesi, "Life Prediction and Reliability Analysis of Ceramic Structures Under Combined Static and Cyclic Fatigue", ASME paper 98-GT-569, 1998
53. D.G. Robinson, "A Survey of Probabilistic Methods Used in Reliability, Risk and Uncertainty Analysis: Analytical Techniques I", SAND98-1189, Sandia National Laboratories, 1998
54. G. Rogers (Boeing), "Probabilistic Approaches-Where Do We Go From Here?", Key-note speech PMC 2001 Conference, Newport Beach, California, USA.
55. F.M. Safie and R.T. Hage, "A Simulation Model for Risk Assessment of Turbine Wheels", Annual Proceedings Reliability and Maintainability Symposium, 1991
56. A. Saith, P.F. Norton and V.M. Parthasarathy, "Application of SPSLIFE to preliminary Design Evaluation and Life Assessment of CSGT Components", *Transactions of the ASME*, Vol. 117, pp 424-431, July 1995
57. J.A. Salem, N. Nemeth, L.M. Powers and S.R. Choi, "Reliability Analysis of Uniaxially Ground Brittle Materials", NASA-TM-106852, 1995

58. A.R. Shah, M.C. Shiao, V.K. Nagpal and C.C. Chamis, "Probabilistic Evaluation of Uncertainties and Risk on Aerospace Components", NASA-TM-105603, Lewis Research Centre, 1992
59. N.D. Singpurwalla, "An Interactive PC-Based Procedure for Reliability Assessment Incorporating Expert Opinion and Survival Data", *Journal of the American Statistical Association*, Vol.83, No.401, March 1988
60. Southwest Research Institute "Probabilistic Structural Analysis Methods (PSAM) for Select Space Propulsion System Components, Contractor's Annual Report", NASA/CR-1999-208872, 1999
61. G. Sturmer, A. Schulz and S.Wittig, "Lifetime Prediction for Ceramic Gas Turbine Components", *Transactions of the ASME*, Vol. 115, pp 70-75, January 1993
62. G.S. Szekely, W.H. Teichert, C.E. Brenner, H.J. Pradlwarter, M. Klein and G. Schueller, "Practical Procedures for Reliability Estimation of Spacecraft Structures and Their Components", *AIAA Journal*, Vol.36, No. 8, August 1998
63. D.J. Thomas and R.C. Wetherhold, "Reliability Analysis of Ceramic Matrix Composite Laminates", *Journal of Engineering for Gas Turbines and Power*, Vol. 115, pp 117-121, January 1993
64. C. Tong and J. Kappas, "Risk Assessment of the TF30 Engine Second and Third Stage Compressor Discs", DSTO-Technical Report, to be published in 2002
65. R.G. Tryon and T.A. Cruse "A reliability-based model to predict scatter in fatigue crack nucleation life", *Fatigue & Fracture of Engineering Materials and Structures*, Vol. 21, pp 257-267, 1988
66. UK DEF STAN 00-971, General Specifications For Aircraft Gas Turbine Engines, 1987
67. USAF MIL-STD-1783, Engine Structural Integrity Program (ENSIP), 1984
68. US MIL-STD-882C, Military Standard System Safety Program Requirements, 1993
69. P.W. Whaley, "A Probabilistic Framework for the Analysis of Corrosion Damage in Aging Aircraft", AIAA-98-1936, 1998
70. Y.T. Wu and K.S. Chen, "Characterisation of Input Uncertainties Using Random Variables for Reliability Analysis", AIAA Forum on Non-Deterministic Approaches Conference, AIAA 99-1606, 1999
71. Y.T. Wu, H.R. Millwater, M.P.Enright, "Probabilistic Methods for DARWIN", 4th Annual FAA/USAF Workshop *Application of Probabilistic Methods to Gas Turbine Engines*, Dec 7-8, Florida, 1999
72. Y.T. Wu, M.P.Enright, H.R. Millwater, G.Chell, C.Kuhlman, G.R. Leverant, "Probabilistic Methods for Design Assessment of Reliability with Inspection", Southwest Research Institute", 2000

73. Y. Xiaoming, H.C. Kuang and K.C. Kyung, "Probabilistic Structural Durability Prediction", AIAA Journal, Vol. 36, No. 4, April 1998
74. J.N. Yang, G.C. Salivar and C.G. Annis Jr, "Statistical Modeling of Fatigue-Crack Growth in a Nickel-Base Superalloy", Engineering Fracture Mechanics Vol. 18, No. 2, pp257-270, 1983
75. E.V. Zaretsky, "MMC Life System Development (Phase I)-A NASA/Pratt & Whitney Life Prediction Cooperative Program", NASA Reference Publication 1361, 1996

Appendix 1

Series Reliability Model for Engine Components

Given an engine component can be broken down into a number of zones or elements n , the component survivability, S_c , is given by the product of the individual survivabilities for each zone or element, S_i :

$$S_c = \prod_{i=1}^n S_i \quad (1)$$

(equation 15, section 6.1)

The probability of failure of the component is given by:

$$P_c = 1 - S_c \quad (2)$$

Substituting for probability of survival of each element and assuming that the failure events are independent, (2) becomes:

$$P_c = 1 - \prod_{i=1}^n S_i \quad (2)'$$

$$P_c = 1 - \prod_{i=1}^n (1 - P_i) \quad (3)$$

For $n=3$, to simplify the example, (3) gives when expanded:

$$P_c = 1 - (1 - P_1) * (1 - P_2) * (1 - P_3) = 1 - [1 - P_1 - P_2 + P_1P_2 - P_3 + P_2P_3 + P_1P_3 - P_1P_2P_3] \quad (4)$$

Assuming that P_1, P_2, P_3 are small, their product will be negligible and (4) simplifies to:

$$P_c \approx P_1 + P_2 + P_3 \quad (5)$$

For n zones or elements, the probability of failure of a component is therefore given by:

$$P_c = \sum_{i=1}^{i=n} P_i \quad (6)$$

(equation 16, section 6.1)

A thorough study of engine system risk issues and models are currently undertaken in DSTO and a report will be available by end of 2002 [Kappas and Antoniou, 2002].

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Joanna Kappas

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2. TITLE Review of Risk and Reliability Methods for Aircraft Gas Turbine Engines				3. SECURITY CLASSIFICATION (FOR UNCLASSIFIED REPORTS THAT ARE LIMITED RELEASE USE (L) NEXT TO DOCUMENT CLASSIFICATION) Document (U) Title (U) Abstract (U)					
4. AUTHOR(S) Joanna Kappas				5. CORPORATE AUTHOR Aeronautical and Maritime Research Laboratory 506 Lorimer St Fishermans Bend Victoria 3207 Australia					
6a. DSTO NUMBER DSTO-TR-1306		6b. AR NUMBER AR-012-320		6c. TYPE OF REPORT Technical Report		7. DOCUMENT DATE May 2002			
8. FILE NUMBER M1/9/902		9. TASK NUMBER 99/161		10. TASK SPONSOR DAIRENG		11. NO. OF PAGES 41		12. NO. OF REFERENCES 75	
13. URL on the World Wide Web http://www.dsto.defence.gov.au/corporate/reports/DSTO-TR-1306.pdf						14. RELEASE AUTHORITY Chief, Airframes and Engines Division			
15. SECONDARY RELEASE STATEMENT OF THIS DOCUMENT <i>Approved for public release</i>									
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16. DELIBERATE ANNOUNCEMENT No Limitations									
17. CITATION IN OTHER DOCUMENTS Yes									
18. DEFTTEST DESCRIPTORS Gas turbine engines, Aircraft engines, Reliability (engineering), Risk assessment, Military aircraft, Probability theory									
19. ABSTRACT Risk and reliability assessment of aircraft gas turbine engines for the evaluation of component failure has received increasing interest in the last few years, fuelled by the greater appreciation of stochastic models and the concern for airworthiness issues. This report reviews the current status of probabilistic methods available for the risk and reliability assessment of gas turbine engines and the potential benefits of their implementation in the military environment. The definition of acceptable risk of failure in the military standards and the current relevant activities in the U.S which are of particular interest to the RAAF, are also discussed.									